Introduction to Vectors

Definitions

A *vector* is a mathematical entity that has *magnitude* and *direction*.
Two vectors are *equal to each other* if an only if their magnitudes and directions are the same.
A vector with magnitude equal to zero is called the *zero vector* or the *null vector*.
The vector $-\vec{v}$ has the same magnitude as $\vec{v}$ but has the opposite direction.
A *unit vector* has magnitude equal to 1. Unit vectors in different directions are different from each other.

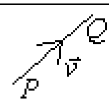
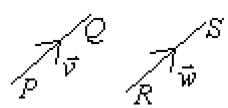
Note that there is a class of vectors known as *localized vectors* where a *point of application* has to be specified in addition to magnitude and direction. We shall not consider such vectors here.

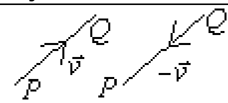
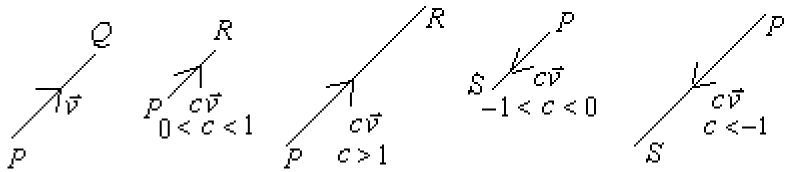
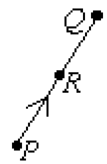
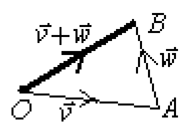
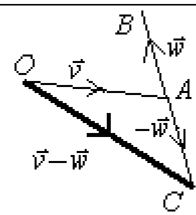
Names: We use letters in bold type or with arrows on top to denote vectors. E.g., we may use ***a***, ***b***, $\vec{u}$, $\vec{v}$**.**

Notation: We shall write the magnitude of vector $\vec{v}$ as $\|\vec{v}\|$

When coordinate systems in $\Re^2$ or $\Re^3$ are used, a vector is said to be in *standard position* when the initial point of the vector coincides with the origin O. Then, the components of the vector are the coordinates of the terminal point of the vector.

Some basic properties of vectors in geometry, in $\Re^2$, and in $\Re^3$ are given in the following table:

Property	Geometry	$\Re^2$	$\Re^3$
Representation	We represent a vector by a directed line segment or a displacement. E.g., $\vec{v} = \overrightarrow{PQ}$. 	$\vec{v} = \langle v_1, v_2 \rangle$. If $\vec{v} = \overrightarrow{OA}$, where O is the origin, then the coordinates of A are (v_1, v_2).	$\vec{v} = \langle v_1, v_2, v_3 \rangle$. If $\vec{v} = \overrightarrow{OA}$, where O is the origin, the coordinates of A are (v_1, v_2, v_3).
Magnitude	If $\vec{v} = \overrightarrow{PQ}$, then $\|\vec{v}\|$ = length of line segment PQ.	$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$	$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
Direction	If $\vec{v} = \overrightarrow{PQ}$, the direction of $\vec{v}$ is the direction from P to Q.	The direction of $\vec{v}$ is the direction from the point (0, 0) to (v_1, v_2).	The direction of $\vec{v}$ is the direction from the point (0, 0, 0) to (v_1, v_2, v_3).
Equality	If $\vec{v} = \overrightarrow{PQ}$ and $\vec{w} = \overrightarrow{RS}$, then $\vec{v} = \vec{w}$ if and only if PQ is equal and parallel to RS. 	If $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$, then $\vec{v} = \vec{w}$ if and only if $v_1 = w_1$ and $v_2 = w_2$.	If $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$, then $\vec{v} = \vec{w}$ if and only if $v_1 = w_1$, $v_2 = w_2$ and $v_3 = w_3$.
Zero Vector	Geometrically, a zero vector may be represented by a point.	$\boldsymbol{O} = \langle 0, 0 \rangle$	$\boldsymbol{O} = \langle 0, 0, 0 \rangle$

Property	**Geometry**	$\Re^2$	$\Re^3$
The vector $-\vec{v}$	If $\vec{v} = \overrightarrow{PQ}$, then $-\vec{v} = \overrightarrow{QP}$. 	$-\vec{v}$ = <$-v_1$, $-v_2$>.	$-\vec{v}$ = <$-v_1$, $-v_2$, $-v_3$>
The vector $c\,\vec{v}$, where c is a scalar For all values of c we have $\lVert c\vec{v} \rVert = \lvert c \rvert \lVert \vec{v} \rVert$.	If $c = 0$, then $c\,\vec{v}$ is the zero vector. If $c > 0$, then $c\,\vec{v} = \overrightarrow{PR}$, where R is on PQ such that $PR = c\,PQ$. Note that Q and R are on the same side of P. If $c < 0$, then $c\,\vec{v} = \overrightarrow{PS}$, where S is on QP such that $PS = \lvert c \rvert\,PQ$. Note that Q and S are on opposite sides of P. 	$c\,\vec{v}$ = <cv_1, cv_2>.	$c\,\vec{v}$ = <cv_1, cv_2, cv_3>
Unit vector The unit vector in the direction of the vector $\vec{v}$ is given by $\vec{u} = \dfrac{\vec{v}}{\lVert \vec{v} \rVert}$.	If $\vec{v} = \overrightarrow{PQ}$, the unit vector $\vec{u}$ in the direction of $\vec{v}$ is given by $\vec{u} = \overrightarrow{PR}$, where R is a point on PQ with length $PR = 1$. R must be on the same side of P as Q. 	If $\vec{v}$ = <v_1, v_2>, then $\vec{u} = \dfrac{1}{\sqrt{v_1^2 + v_2^2}} < v_1, v_2 >$.	If $\vec{v}$ = <v_1, v_2, v_3>, then $\vec{u} = \dfrac{1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} < v_1, v_2, v_3 >$.
Addition of Vectors	Let $\vec{v}$ and $\vec{w}$ be two given vectors. To add these, we draw line segments OA and AB so that $\overrightarrow{OA} = \vec{v}$ and $\overrightarrow{AB} = \vec{w}$. Then, $\vec{v} + \vec{w} = \overrightarrow{OB}$. 	If $\vec{v}$ = <v_1, v_2> and $\vec{w}$ = <w_1, w_2>, then $\vec{v} + \vec{w}$ = < $v_1 + w_1$, $v_2 + w_2$>	If $\vec{v}$ = <v_1, v_2, v_3> and $\vec{w}$ = <w_1, w_2, w_3>, then $\vec{v} + \vec{w}$ = <$v_1 + w_1$, $v_2 + w_2$, $v_3 + w_3$>
Subtraction of Vectors $\vec{v} - \vec{w}$ is defined to be $\vec{v}$ +($-\vec{w}$)	Let $\vec{v}$ and $\vec{w}$ be two given vectors. To obtain $\vec{v} - \vec{w}$, we draw line segments OA and AC so that $\overrightarrow{OA} = \vec{v}$ and $\overrightarrow{AC} = -\vec{w}$. Then, $\vec{v} - \vec{w} = \overrightarrow{OC}$. 	If $\vec{v}$ = <v_1, v_2> and $\vec{w}$ = <w_1, w_2>, then $\vec{v} - \vec{w}$ = < $v_1 - w_1$, $v_2 - w_2$>	If $\vec{v}$ = <v_1, v_2, v_3> and $\vec{w}$ = <w_1, w_2, w_3>, then $\vec{v} - \vec{w}$ = <$v_1 - w_1$, $v_2 - w_2$, $v_3 - w_3$>